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Maxwell Chern-Simons solitons from type IIB string theory

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We study various three-dimensional supersymmetric Maxwell Chern-Simons solitons by using type IIB brane configurations. We give a systematic classification of soliton spectra such as topological BPS vortices and nontopological vortices in an $\mathcal{N}=2,3$ supersymmetric Maxwell Chern-Simons system via the branes of type IIB string theory. We identify the brane configurations with the soliton spectra of the field theory and obtain nice agreement with field theory aspects. We also discuss possible brane constructions for BPS domain wall solutions. [S0556-2821(99)01118-2]

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I. INTRODUCTION

The recent developments in nonperturbative string theories have provided new powerful tools to understand supersymmetric gauge theories [1]. The low-energy dynamics of the D-branes is described by supersymmetric gauge theories which can be related to the ground-state excitations of fundamental strings connecting pairs of D-branes [2]. The Bogomol'nyi-Prasad-Sommerfield (BPS) brane configurations in the background led to many exact results on the vacuum structure of supersymmetric gauge theories.

The novel aspects of three-dimensional supersymmetric gauge theories can be understood via type IIB brane configurations, in which D3-branes are suspended between two Neveu-Schwarz five-branes (NS5-branes) [3,4]. This construction gives an explanation of mirror symmetry in three dimensions via SL(2,**Z**) duality of type IIB string theory. This mirror symmetry is also true for BPS vortices and exchanges particles and vortices [5,6].

Recently three-dimensional gauge theories have been studied and classified by using more general type IIB brane configurations, in which D3-branes are suspended between an NS5-brane and a (p,q)5-brane [7]. In these brane configurations, three-dimensional field theories, in general, turned out to be supersymmetric Maxwell Chern-Simons gauge theories with $\mathcal{N}=4$, 3, 2, 1 supersymmetry. $\mathcal{N}=4$ supersymmetry can only be realized in the NS5-D3-NS5 configuration, which gives supersymmetric QED without a Chern-Simons term. The NS5-D3-(p,q)5 configuration gives $\mathcal{N}=3$, 2, 1 supersymmetric Maxwell Chern-Simons theory, where the $\mathcal{N}=3$ case [8,9] is not much known although it is interesting.

$\mathcal{N}=2$ supersymmetric Maxwell Chern-Simons theory is comparably well known and its soliton solutions have been considerably studied over the years [10–13]. Whereas the Maxwell-Higgs model supports only electrically neutral vortices as topologically stable soliton solutions [14], the addition of the Chern-Simons term gives rise to topologically stable solutions that are electrically charged and carry magnetic flux and nonzero angular momentum [15,16]. In this theory, there exist topological as well as nontopological BPS multisoliton solutions since, in three dimensions, the superpotential allows symmetry broken and unbroken vacua [11]. Thus, there can be a peculiar solution known as the (topological) domain wall which is a one-dimensional object in three dimensions [16,17]. In crossing the domain wall, the vacua are different on two sides. In addition, it is known that there can also be (nontopological) domain walls residing in the symmetric phases [16,17]. In this paper we will discuss how these kinds of topological objects can be described in terms of the above brane configurations.

The organization of the paper is as follows. In Sec. II, by using a similar method as in Refs. [7,18], we classify BPS configurations consisting of relatively rotated two M5-branes with N_c M2-branes in between and N_f M5-branes as well as other M2-branes corresponding to solitons in three dimensions. We identify possible supersymmetry for each M-brane configuration. These are then transformed to the brane configurations in type IIB string theory after compactifying the M-theory and then applying T-duality. This construction will provide the classification of all possible BPS solitons such as topological and nontopological BPS vortices in three-dimensional supersymmetric Maxwell Chern-Simons system. Although there is a BPS M2-brane which may be a

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plausible candidate for the BPS domain wall constructed in field theory [16,17], we have not been able to identify an appropriate configuration for a finite energy density solution. In Sec. III, we study topological BPS vortices as well as nontopological vortices by using the M-brane configurations constructed in Sec. II. We compare the brane configurations with the soliton spectra already known from field theories and obtain a nice agreement with field theory results. In Sec. IV, we summarize our main results and give qualitative arguments on the topological as well as nontopological BPS domain wall solutions. We also discuss mirror symmetry for the solitons, and indicate future directions.

After the completion of this paper, we were informed from Ohta [19] that he also independently arrived at similar results on the moduli space of vacua in the theories considered here.

II. BPS BRANE CONFIGURATIONS

The authors of Ref. [7] examined three-dimensional gauge dynamics by using type IIB brane configurations. They obtained these from the M-theory configurations of M2-branes suspended between two M5-branes at angles [18]. The BPS brane configurations in supersymmetric M-brane backgrounds can be obtained by the following intersection rules [20]: In the M2-brane background, an M2-brane probe can preserve 1/4 supersymmetry only without overlap and an M5-brane probe can only do so in the string intersection. In the M5-brane background, an M5-brane probe can preserve 1/4 supersymmetry only in string or three brane intersections and an M2-brane probe can only do so in the string intersection. These situations give various brane configurations and residual supersymmetries.

In this section, we construct the brane configurations corresponding to three-dimensional gauge theories with soliton solutions. For this purpose we need to count the number of supersymmetries remaining in the brane configurations. The cases of our interest are realized by inserting other M2-branes intersecting the $\widetilde{\mathrm{M5}}$-branes that give rise to the hypermultiplets. After compactification on $x^\natural$ (the symbol $\natural$ indicates the 11th direction, 10) and T_2-duality (the subscript 2 stands for the direction of T-duality) of the brane configurations, these are reduced to the type IIB brane configurations. The M2-branes are transformed to a D1-brane or a D3-brane which correspond to the vortex or domain wall solutions, respectively. We now explain each case separately.

First let us consider an M2-brane and $\widetilde{\mathrm{M5}}$-branes between two M5-branes with relative angles and another $\widetilde{\mathrm{M2}}$-brane in the directions x^2 and x^a (the superscript a indicates one direction out of 7, 8, and 9 according to the intersection rules), which corresponds to the D1($0a$) string embedded in D5(012789)-brane in type IIB string theory. The world volumes of these branes are given by

$$\mathrm{M5}:(012345),$$

$$\mathrm{M2}:(01|6|),$$

$$\mathrm{M5}':\left(01\begin{bmatrix}2\\ \natural\end{bmatrix}_\theta\begin{bmatrix}3\\ 7\end{bmatrix}_\psi\begin{bmatrix}4\\ 8\end{bmatrix}_\varphi\begin{bmatrix}5\\ 9\end{bmatrix}_\rho\right), \tag{1}$$

$$\widetilde{\mathrm{M5}}:(01789\natural),$$

$$\widetilde{\mathrm{M2}}:(02a),$$

where the vertical line in the M2-brane world volume denotes that the sixth direction is bounded by the two M5-branes, and the vertical arrays in the second M5′-brane world volume indicate that the brane is rotated along the planes by the indicated angles. The $\widetilde{\mathrm{M2}}$ is necessary for our purpose but was not in Ref. [7]. These branes impose the following constraints on the Killing spinor ϵ [18]

$$\mathrm{M5}:\Gamma_{012345}\epsilon=\epsilon, \tag{2}$$

$$\mathrm{M2}:\Gamma_{016}\epsilon=\epsilon, \tag{3}$$

$$\mathrm{M5}':R\Gamma_{012345}R^{-1}\epsilon=\epsilon, \tag{4}$$

$$\widetilde{\mathrm{M2}}:\Gamma_{02a}\epsilon=\epsilon, \tag{5}$$

$$\widetilde{\mathrm{M5}}:\Gamma_{01789\natural}\epsilon=\epsilon, \tag{6}$$

where the rotation matrix for the second M5′-brane is parametrized by the four angles as follows:

$$R=\exp\left\{\frac{\theta}{2}\Gamma_{2\natural}+\frac{\psi}{2}\Gamma_{37}+\frac{\varphi}{2}\Gamma_{48}+\frac{\rho}{2}\Gamma_{59}\right\}. \tag{7}$$

Since $\Gamma_{012\cdots9\natural}=1$ and so $\Gamma_{01789\natural}=\Gamma_{016}\Gamma_{012345}$, the condition (6) is a redundant one. So we must solve just Eqs. (2)–(5) simultaneously as functions of the four angles θ,ψ,φ, and ρ. Since $R\Gamma_{012345}R^{-1}=R^2\Gamma_{012345}$, Eq. (4) becomes

$$(R^2-1)\epsilon=0. \tag{8}$$

By a straightforward calculation, we obtain

$$R^2-1=2R\Gamma_{2\natural}\left\{\sin\frac{\theta}{2}\cos\frac{\psi}{2}\cos\frac{\varphi}{2}\cos\frac{\rho}{2}-\Gamma_{2\natural 37}\cos\frac{\theta}{2}\sin\frac{\psi}{2}\cos\frac{\varphi}{2}\cos\frac{\rho}{2}-\Gamma_{2\natural 48}\cos\frac{\theta}{2}\cos\frac{\psi}{2}\sin\frac{\varphi}{2}\cos\frac{\rho}{2}\right.$$
$$-\Gamma_{2\natural 59}\cos\frac{\theta}{2}\cos\frac{\psi}{2}\cos\frac{\varphi}{2}\sin\frac{\rho}{2}+\Gamma_{3748}\sin\frac{\theta}{2}\sin\frac{\psi}{2}\sin\frac{\varphi}{2}\cos\frac{\rho}{2}+\Gamma_{4859}\sin\frac{\theta}{2}\cos\frac{\psi}{2}\sin\frac{\varphi}{2}\sin\frac{\rho}{2}$$
$$\left.+\Gamma_{3759}\sin\frac{\theta}{2}\sin\frac{\psi}{2}\cos\frac{\varphi}{2}\sin\frac{\rho}{2}-\Gamma_{2\natural 374859}\cos\frac{\theta}{2}\sin\frac{\psi}{2}\sin\frac{\varphi}{2}\sin\frac{\rho}{2}\right\}. \tag{9}$$

The gamma matrices appearing in the spinor constraints commute with each other except Γ_{02a}. Since the square of the matrices is unity and the traces of their products vanish, we can arrange these matrices by the same method as in Refs. [7,18] in the following forms:

$$\Gamma_{012345}=\mathrm{diag}(\mathbf{1}_{16},-\mathbf{1}_{16}),$$
$$\Gamma_{2\natural 37}=\mathrm{diag}(\mathbf{1}_{8},-\mathbf{1}_{8},\ldots),$$
$$\Gamma_{2\natural 48}=\mathrm{diag}(\mathbf{1}_{4},-\mathbf{1}_{4},\mathbf{1}_{4},-\mathbf{1}_{4},\ldots),$$
$$\Gamma_{2\natural 59}=\mathrm{diag}(\mathbf{1}_{2},-\mathbf{1}_{2},\mathbf{1}_{2},-\mathbf{1}_{2},\mathbf{1}_{2},-\mathbf{1}_{2},\mathbf{1}_{2},-\mathbf{1}_{2},\ldots), \tag{10}$$

where $\mathbf{1}_n$ denotes the $n\times n$ identity matrix, and the rest of Eq. (9) is determined by the products of the above matrices. Since $\Gamma_{012\cdots 9\natural}=1$, Γ_{016} is also determined by the products of the gamma matrices in Eq. (10) as

$$\Gamma_{016}-1=-2\times\mathrm{diag}(\mathbf{0}_2,\mathbf{1}_2,\mathbf{1}_2,\mathbf{0}_2,\mathbf{1}_2,\mathbf{0}_2,\mathbf{0}_2,\mathbf{1}_2,\ldots). \tag{11}$$

On the gamma matrix basis (10), we have the following expression:

$$R^2-1=2R\Gamma_{2\natural}\times\mathrm{diag}\left(\sin\left(\frac{\theta-\psi-\varphi-\rho}{2}\right)\mathbf{1}_2,\sin\left(\frac{\theta-\psi-\varphi+\rho}{2}\right)\mathbf{1}_2,\right.$$
$$\sin\left(\frac{\theta-\psi+\varphi-\rho}{2}\right)\mathbf{1}_2,\sin\left(\frac{\theta-\psi+\varphi+\rho}{2}\right)\mathbf{1}_2,$$
$$\sin\left(\frac{\theta+\psi-\varphi-\rho}{2}\right)\mathbf{1}_2,\sin\left(\frac{\theta+\psi-\varphi+\rho}{2}\right)\mathbf{1}_2,$$
$$\left.\sin\left(\frac{\theta+\psi+\varphi-\rho}{2}\right)\mathbf{1}_2,\sin\left(\frac{\theta+\psi+\varphi+\rho}{2}\right)\mathbf{1}_2,\ldots\right). \tag{12}$$

Considering the above expression of Γ_{016}, the remaining supersymmetry is now determined by the sin functions of the four angles in the first, fourth, sixth, and seventh blocks of the R^2-1 matrix. We summarize in Table I the BPS brane configurations at angles and the various supersymmetric theories in three dimensions obtained in Ref. [7] in the absence of the $\widetilde{\mathrm{M2}}$, where we indicated only one representative in each case since (3,7)-, (4,8)-, and (5,9)-planes are on an equal footing with each other.

Now we consider the BPS configurations constructed by M2-branes such as $\widetilde{\mathrm{M2}}$ in the M-brane background (1). We require that Eq. (5) should not completely break supersymmetry. Note that the simultaneous solution to Eqs. (5) and (8) can induce a new constraint depending on their commutativity. One solution is obtained when the gamma matrices appearing in Eq. (9) all commute with Γ_{02a} and we have no further constraint. The other is when the gamma matrices in Eq. (9) do not commute with Γ_{02a}, for which we have an additional condition on the spinor ϵ:

$$[R^2,\Gamma_{02a}]\epsilon=0. \tag{13}$$

Of course, in this case, the gamma matrices Γ_{02a} cannot be simultaneously diagonalized.

Let us consider the first case. From expression (9), we see that we must put at least two angles to zero, resulting in two-angle cases. There are six possibilities for the gamma matrices to commute with Γ_{02a}. By examining each case separately, the a direction is uniquely determined. The result is the following:

TABLE I. Brane configurations at angles and various supersymmetric theories in three dimensions.

	Angles	Condition	SUSY	$d=3$	M5′
1	$\theta(2\natural)$	$\theta=0$	$\frac{1}{4}$	$\mathcal{N}=4$	NS5(12345)
2(i)	$\varphi(48),\rho(59)$	$\rho=\varphi$	$\frac{1}{8}$	$\mathcal{N}=2$	$\mathrm{NS5}(123[\frac{4}{8}]_\varphi[\frac{5}{9}]_\varphi)$
2(ii)	$\theta(2\natural),\rho(59)$	$\rho=\theta$	$\frac{1}{8}$	$\mathcal{N}=2$	$(p,q)5(1234[\frac{5}{9}]_\theta)$
3(i)	$\psi(37),\varphi(48),\rho(59)$	$\rho=\psi+\varphi$	$\frac{1}{16}$	$\mathcal{N}=1$	$\mathrm{NS5}(12[\frac{3}{7}]_\psi[\frac{4}{8}]_\varphi[\frac{5}{9}]_{\psi+\varphi})$
3(ii)	$\theta(2\natural),\varphi(48),\rho(59)$	$\rho=\theta+\varphi$	$\frac{1}{16}$	$\mathcal{N}=1$	$(p,q)5(123[\frac{4}{8}]_\varphi[\frac{5}{9}]_{\theta+\varphi})$
4(i)		$\theta=\psi-\varphi-\rho$	$\frac{1}{16}$	$\mathcal{N}=1$	$(p,q)5(12[\frac{3}{7}]_\psi[\frac{4}{8}]_\varphi[\frac{5}{9}]_{\psi-\varphi-\theta})$
4(ii)	$\theta(2\natural),\psi(37),\varphi(48),\rho(59)$	$\theta=-\rho,\psi=\varphi$	$\frac{1}{8}$	$\mathcal{N}=2$	$(p,q)5(12[\frac{3}{7}]_\varphi[\frac{4}{8}]_\varphi[\frac{5}{9}]_{-\theta})$
4(iii)		$\theta=\psi=\varphi=-\rho$	$\frac{3}{16}$	$\mathcal{N}=3$	$(p,q)5(12[\frac{3}{7}]_\theta[\frac{4}{8}]_\theta[\frac{5}{9}]_{-\theta})$

$$\theta=\psi=0 \quad (a=7), \qquad \theta=\varphi=0 \quad (a=8),$$
$$\theta=\rho=0 \quad (a=9), \qquad \psi=\varphi=0 \quad (a=9),$$
$$\psi=\rho=0 \quad (a=8), \qquad \varphi=\rho=0 \quad (a=7). \tag{14}$$

As an example, let us consider the $\theta=\psi=0$ $(a=7)$ case. In order to find the solution, it is convenient to choose maximally diagonalized basis different from Eqs. (10) and (11):

$$\Gamma_{012345}=\mathrm{diag}(\mathbf{1}_{16},-\mathbf{1}_{16}),$$
$$\Gamma_{4859}=\mathrm{diag}(\mathbf{1}_8,-\mathbf{1}_8,\ldots),$$
$$\Gamma_{016}=\mathrm{diag}(\mathbf{1}_4,-\mathbf{1}_4,\mathbf{1}_4,-\mathbf{1}_4,\ldots),$$
$$\Gamma_{027}=\mathrm{diag}(\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\ldots). \tag{15}$$

Using Eqs. (15), we can rewrite Eq. (9) as

$$R^2-1=2R\Gamma_{48}\,\mathrm{diag}\left(\sin\frac{\varphi-\rho}{2}\mathbf{1}_8,\sin\frac{\varphi+\rho}{2}\mathbf{1}_8,\ldots\right). \tag{16}$$

On this basis, the first condition (2) destroys the second 16 components of the Killing spinor, and we have to examine the conditions (3) and (8) for the first 16 components. For $\varphi=\rho$ the remaining supersymmetry is reduced by $\frac{1}{2}$ compared with the configuration without the $\widetilde{\mathrm{M2}}$-branes [case 2(i) in Table I) and this brane will correspond to a BPS state (a vortex) in three dimensions.

Another example is to choose the angles as $\psi=\varphi=0$ $(a=9)$. We can again arrange the matrices as

$$\Gamma_{012345}=\mathrm{diag}(\mathbf{1}_{16},-\mathbf{1}_{16}),$$
$$\Gamma_{2\natural 59}=\mathrm{diag}(\mathbf{1}_8,-\mathbf{1}_8,\ldots),$$
$$\Gamma_{016}=\mathrm{diag}(\mathbf{1}_4,-\mathbf{1}_4,\mathbf{1}_4,-\mathbf{1}_4,\ldots),$$
$$\Gamma_{029}=\mathrm{diag}(\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\mathbf{1}_2,-\mathbf{1}_2,\ldots), \tag{17}$$

Using Eqs. (17), we can rewrite Eq. (9) as

$$R^2-1=2R\Gamma_{2\natural}\,\mathrm{diag}\left(\sin\frac{\theta-\rho}{2}\mathbf{1}_8,\sin\frac{\theta+\rho}{2}\mathbf{1}_8,\ldots\right). \tag{18}$$

For $\theta=\rho$ this case also reduces the supersymmetry by $\frac{1}{2}$ compared with the configuration without the $\widetilde{\mathrm{M2}}$-brane [case 2(ii) in Table I] and this also gives a BPS state (a vortex) in three dimensions. For the remaining cases in Eq. (14), we also obtain similar vortex solutions.

Next we consider the second case. Our interest is in the four-angle cases corresponding to $\mathcal{N}=2$ or $\mathcal{N}=3$ supersymmetry [cases 4(ii) and 4(iii) in Table I) because BPS states are possible for these cases. For definiteness, let us choose $a=9$ and the rotation matrix R^2 as

$$R^2=\exp\{\theta(\Gamma_{2\natural}-\Gamma_{59})+\psi(\Gamma_{37}+\Gamma_{48})\}. \tag{19}$$

By a straightforward calculation, Eq. (13) becomes

$$[R^2,\Gamma_{029}]\epsilon=\Gamma_{029}(e^{-4\theta\Gamma_{2\natural}}-1)R^2\frac{1+\Gamma_{2\natural 59}}{2}\epsilon=0, \tag{20}$$

which reduces to the following equation:

$$\frac{1+\Gamma_{2\natural 59}}{2}\epsilon=0 \tag{21}$$

if[1] $\theta\neq n\pi/2$ $(n\in\mathbf{Z})$. In the four-angle case 4(ii), one can directly check using Eqs. (10) and (11) that, if the constraints (2), (3), and (8) are imposed on the spinor ϵ, the number of unbroken supersymmetries is 4 $(\mathcal{N}=2)$. The condition (21) does not produce further constraints. Interestingly, however, the $\mathcal{N}=3$ supersymmetry [$\theta=\psi$, case 4(iii) in Table I] is

[1]In the case of $\theta=\psi$, the M5′-brane is parallel to the $\widetilde{\mathrm{M5}}$-brane in the limit $\theta\rightarrow\pi/2$ corresponding to $\kappa=-(1/g_s)\tan\theta=-p/q\rightarrow\infty$. When $\kappa\rightarrow\infty$, the vector multiplet decouples and the theory becomes a theory of a free massless hypermultiplet with $\mathcal{N}=4$ supersymmetry [6]. It turns out that the supersymmetric pure Chern-Simons system discussed in Refs. [11,8] indeed corresponds to another limit, i.e., $L_6\rightarrow 0$ with κ fixed, where L_6 is the length of the D3-brane in the x^6-direction. Thus we consider here only the case of $\theta\neq\pi/2$.

further broken to $\mathcal{N}=2$ by the condition (21). Since the gamma matrix Γ_{02a} squares to unity and is traceless, its eigenvalues must be ± 1 and the multiplicities of these eigenvalues should be the same. Moreover, since the traces of products with the gamma matrices in the spinor constraints vanish, the condition (5) further breaks supersymmetry at least by half. Consequently, the condition (5) maximally preserves the $\mathcal{N}=1$ supersymmetry and so the $\widetilde{\text{M2}}$-brane will correspond to a BPS state (a vortex) in $\mathcal{N}=2$ or $\mathcal{N}=3$ supersymmetric theory. This is consistent with the field theory results in [8,9,11,13].

For other cases with the rotation matrix different from Eq. (19), the direction a of a $\widetilde{\text{M2}}$-brane should be differently chosen. For example, if we take

$$R^2=\exp\{\theta(\Gamma_{2\natural}-\Gamma_{37})+\psi(\Gamma_{48}+\Gamma_{59})\}, \tag{22}$$

then $a=7$ and similar solutions can be obtained.

For the purpose of constructing nontopological vortices, let us consider another $\widehat{\text{M2}}$-brane in the $(b,\natural)$ directions, instead of the $\widetilde{\text{M2}}$-brane in Eq. (1), and then rotate it by the same rotation R in Eq. (7) (b indicates one direction out of 3, 4, and 5 according to the intersection rules). The Killing spinor condition for this brane is given by

$$\widehat{\text{M2}}: R\Gamma_{0b\natural}R^{-1}\epsilon=\epsilon, \tag{23}$$

instead of Eq. (5). Note that the rotation in the planes containing neither b nor $\natural$ does not affect the $\widehat{\text{M2}}$-brane. What we need for our purpose are not F-strings but D-strings or their composites, so the angle θ needs to be nonzero. We now seek simultaneous solutions to Eqs. (2), (3), (8), and (23). For definiteness, let us take $b=5$. Then Eq. (23) is cast into

$$R^2_{\theta\rho}\Gamma_{05\natural}\epsilon=\epsilon, \tag{24}$$

and, from the conditions (8) and (24), we get another constraint

$$(R^4_{\theta\rho}-1)R^2_{\psi\varphi}\Gamma_{05\natural}\epsilon=0, \tag{25}$$

where

$$R^2_{\theta\rho}=\exp(\theta\Gamma_{2\natural}+\rho\Gamma_{59}),\quad R^2_{\psi\varphi}=\exp(\psi\Gamma_{37}+\varphi\Gamma_{48}). \tag{26}$$

Since the matrices $R_{\theta\rho}$, $R_{\psi\varphi}$, and $\Gamma_{05\natural}$ are nonsingular, the condition (25) can be reduced to the following form:

$$(R^2_{\theta\rho}-R^{-2}_{\theta\rho})\epsilon=2\Gamma_{2\natural}(\sin\theta\cos\rho-\cos\theta\sin\rho\Gamma_{2\natural 59})\epsilon=0. \tag{27}$$

In $\mathcal{N}=2$ theory, in which we put rotation angles not involving $\natural$ and b directions to zero and $\theta=-\rho$, the condition (27) is essentially equal to Eq. (8) and so a redundant one. Using Eq. (8) or (27), the spinor constraint (24) reduces to the following condition:

$$\widehat{\text{M2}}: \Gamma_{05\natural}\epsilon=\epsilon. \tag{28}$$

Of course, for the more general case (23), there are three possibilities for the b direction. The results for each case are the following:

$$\psi=\varphi=0\ \ (b=5),\quad \psi=\rho=0\ \ (b=4),$$
$$\varphi=\rho=0\ \ (b=3), \tag{29}$$

and the condition (28) is generalized to

$$\widehat{\text{M2}}: \Gamma_{0b\natural}\epsilon=\epsilon. \tag{30}$$

If we take the direction of the $\widehat{\text{M2}}$-brane to satisfy the condition (29), all the gamma matrices appearing in the spinor constraints commute with each other and so can be simultaneously diagonalized. We thus see that $\Gamma_{0b\natural}$ can be taken in the same form as Γ_{029} in Eq. (17) and the condition (27) no longer breaks supersymmetry. Thus the $\widehat{\text{M2}}$-brane totally preserves the $\mathcal{N}=1$ supersymmetry, giving a BPS state.

On the other hand, in $\mathcal{N}=3$ theory with $\theta=\psi=\varphi=-\rho$, the condition (27) breaks the supersymmetry from $\mathcal{N}=3$ to $\mathcal{N}=2$ as in topological vortices. Since the condition (28) further breaks the supersymmetry by half, the $\widehat{\text{M2}}$-brane totally preserves the $\mathcal{N}=1$ supersymmetry, again giving a BPS state in $\mathcal{N}=3$ theory. Since the $\widehat{\text{M2}}$-branes will be interpreted as nontopological vortices, the above results are consistent with the fact that nontopological BPS vortices can exist only for $\kappa\neq 0$, i.e., $\theta\neq 0$.

Finally we consider the other possibility of inserting the second M2′-brane along the x^5 and x^9 directions, which corresponds to the D3-brane in type IIB string theory. [Of course, we can also choose the extended directions of the M2′-brane to be (x^3, x^7) or (x^4, x^8) instead of (x^5, x^9) according to the intersection rules.] The condition (5) must be replaced with the condition

$$\text{M2}': \Gamma_{059}\epsilon=\epsilon. \tag{31}$$

Contrary to the case (1), Γ_{059} commutes with the gamma matrices in the spinor constraints. All the gamma matrices can be simultaneously diagonalized and arranged as Eqs. (10) and (11) and

$$\Gamma_{059}=\text{diag}(1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,-1,1,$$
$$-1,\cdots). \tag{32}$$

The supersymmetry is further broken by half by the M2′-brane and so the brane may be a BPS state (a domain wall) in three dimensions.

We have exhausted the M2-brane configurations in the presence of the M-brane background (1) without breaking supersymmetry completely, corresponding to BPS states.[2] In the next section, we will realize the BPS soliton states in three-dimensional field theories in terms of these M-configurations.

III. MAXWELL CHERN-SIMONS VORTICES

In this section we will analyze the Maxwell Chern-Simons vortices via type IIB brane configurations. These in turn can be obtained from the M-brane configurations constructed in Sec. II after compactifying the M-configurations along the 11th direction and then applying T_2-duality [7]. In the process, the number of unbroken supersymmetries is preserved.

From Table I, we see that, if we set one of the four angles θ,ψ,φ,ρ in 3-(i) or 3-(ii) to zero, supersymmetry is enhanced from $\mathcal{N}=1$ to $\mathcal{N}=2$, while, in the 4(i) and 4(ii) cases, it is not enhanced, for example, in the $\theta\to 0$ limit where the Chern-Simons term vanishes. The $\mathcal{N}=3$ case is quite special since, in this case, the four angles should be equal.

As noted in Sec. II, in the zero- and two-angle cases, there is a possibility to introduce the $\widetilde{\mathrm{M2}}$-brane preserving half of the supersymmetry and extended to the $(2a)$-plane. In type IIB string theory, this brane is just the D1-brane along the a-direction and, in three-dimensional field theory, this will correspond to a BPS vortex solution as we will see. In addition, we have shown that the four-angle cases [4(ii) and 4(iii) cases in Table I) also contain the spectrum of supersymmetric BPS vortices. As will be shown, this fact is consistent with the field theory result [13,16,8,9] that the $\mathcal{N}=2$ and $\mathcal{N}=3$ Maxwell Chern-Simons theories admit topological as well as nontopological vortex solutions.

As shown in Ref. [21], the tension of (q_1,q_2)-string in the type IIB metric is

$$T_{(q_1,q_2)}=\frac{1}{2\pi l_s^2}\sqrt{(q_1+q_2\chi)^2+\frac{q_2^2}{g_s^2}}, \tag{33}$$

where χ is a constant background of the type IIB Ramond-Ramond (RR) scalar. In the (p,q)5-brane background, the instanton coupling on the D3-brane world volume induces the Chern-Simons coupling $\kappa=-\chi$ as discussed in Ref. [7]. In this background, the integer charge q_1 is shifted by an arbitrary amount χ due to an analogue of Witten's effect that the electric charge of a monopole is shifted when the theta angle θ is switched on [22]. Thus, although $q_1=0$, a D-string can carry electric charge Q_e proportional to magnetic charge Q_m: $Q_e=|\kappa|Q_m$.

A. Maxwell-Higgs vortices

Hanany and Witten explained the mirror symmetry in three dimensions through the SL(2,**Z**) duality of type IIB superstring [3]. They considered the supersymmetric configuration with N_c D3-branes in (1, 2, 6) directions suspended between two NS5-branes in (1, 2, 3, 4, 5) directions with definite values of the x^6 coordinate. This configuration gives $\mathcal{N}=4$ supersymmetric theory in three dimensions. We can also construct gauge field theories with matter fields if we insert other N_f D5-branes in (1, 2, 7, 8, 9) directions preserving $\mathcal{N}=4$ supersymmetry. These configurations explain $\mathcal{N}=4$ SU(N_c) super Yang-Mill theories with N_f hypermultiplets. This may be generalized by rotating the second NS5$'$-brane by suitable angles.

First let us consider the brane configuration corresponding to case I in Table 1 where two NS5-branes are parallel to each other, i.e., $\theta=\psi=\rho=\varphi=0$. This corresponds to $\mathcal{N}=4$ supersymmetric SU(N_c) gauge theory with N_f hypermultiplets. Here we discuss only the $N_c=1$ and $N_f=1$ case. This configuration is depicted in Fig. 1(a) in which one D3-brane in the direction (126) is suspended between two NS5-branes in (12345) and intersects with a D5-brane in (12789). The same is drawn in Fig. 1(b) when it is seen from the x^6 direction. From this configuration, we get U(1) gauge theory with a massless flavor in the fundamental representation with no Fayet-Iliopoulos (FI) terms.

The $\mathcal{N}=4$ vector multiplet consists of an $\mathcal{N}=2$ real vector multiplet V and a chiral multiplet Φ. (For supersymmetric gauge theories, see, for example, Ref. [23].) In $\mathcal{N}=2$ superspace [12], the vector multiplet V is composed of A_μ ($\mu=0,1,2$), which are the gauge fields on D3-brane world volume, and X_3 which corresponds to the A_3 component of the four-dimensional gauge field. The chiral multiplet Φ contains X_4 and X_5, which correspond to strings describing fluctuations of the D3-brane in the transverse directions $(x^4,\ x^5)$. In addition, there are hypermultiplets consisting of Q and $\tilde{Q}$ in the fundamental representation, which originate from the fundamental strings stretching between the D5- and D3-branes. Using these notations, we can write down the $\mathcal{N}=4$ supersymmetric action in the Coulomb branch:

$$S_{N=4}=\frac{1}{g^2}\left[\int d^3x d^4\theta\Phi^\dagger\Phi+\frac{1}{2}\left(\int d^3x d^2\theta W^\alpha W_\alpha+\text{H.c.}\right)\right]$$
$$+\int d^3x d^4\theta(Q^\dagger e^{2V}Q+\tilde{Q}e^{-2V}\tilde{Q}^\dagger)$$
$$+\frac{1}{\sqrt{2}}\left(\int d^3x d^2\theta\tilde{Q}\Phi Q+\text{H.c.}\right), \tag{34}$$

where W^α is the field strength superfield for the real spinor gauge superfield $U^\alpha(x,\theta)$ and α is the three-dimensional spinor index.

[2]One may also consider an M-wave in 11 dimensions, in which case the Killing spinor condition is $\Gamma_{0\natural}\epsilon=\epsilon$. The M-wave solution may give a D-string in type IIB string theory because it reduces to a D0-brane in type IIA string theory. However, one can see that this solution does not preserve supersymmetry in the M-brane background (1). In this paper, we have not considered M5-brane probes in the M-brane background (1). According to the intersection rules [20], possible M5-brane probes preserving 1/4 supersymmetry are M5$(26ab\natural)$ and M5(26789), where $a,b=7,\ 8,\ 9\ (a\neq b)$.

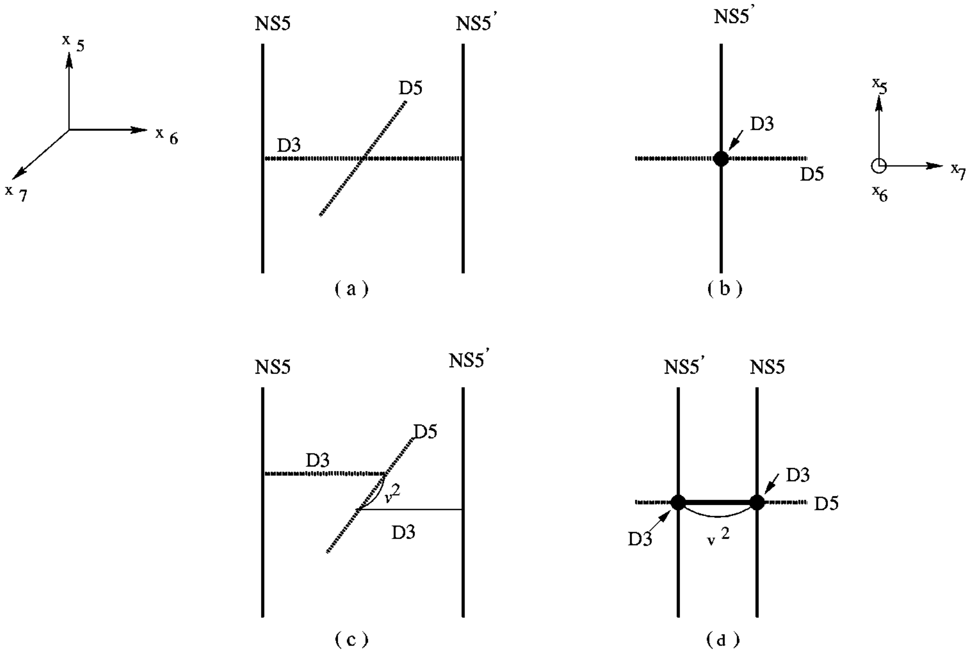

FIG. 1. Topological vortices in Maxwell-Higgs theory. (a) or (b) is for the Coulomb phase and (c) or (d) is a vortex solution in the Higgs phase.

If we turn on FI couplings for the $\mathcal{N}=4$ vector multiplets V and Φ,

$$S_{FI}=-v^2\int d^3xd^4\theta V-\left(\frac{w^2}{\sqrt{2}}\int d^3xd^2\theta\Phi+\text{H.c.}\right), \tag{35}$$

the scalar potential U in the action (34) with Eq. (35) is given by

$$U=\frac{g^2}{2}(|q|^2-|\tilde{q}|^2-v^2)^2+\frac{g^2}{2}|q\tilde{q}-w^2|^2 +(|q|^2+|\tilde{q}|^2)(X_3^2+|\phi|^2). \tag{36}$$

This potential allows only a symmetry broken vacuum:

$$|q|^2-|\tilde{q}|^2=v^2, \quad q\tilde{q}=w^2, \quad X_3=\phi=0. \tag{37}$$

Note that the FI terms come from the relative positions in the (789)-directions of the NS5- and NS5′-branes. The peculiar fact is that, if the FI coupling w for the complex scalar field Φ is nonzero, the hypermultiplet should have all nonzero vacuum expectation values.

Next we consider the brane configuration corresponding to 2-(i) in Table I where the NS5′-brane is at angle $\theta=\psi=0$ and $\rho=\varphi$. This configuration was considered by the authors of Ref. [4] and corresponds to $\mathcal{N}=2$ supersymmetric U(1) gauge theory with a massless flavor in the fundamental representation with no FI terms. This configuration is also depicted in Fig. 1(a) in which one D3-brane in the direction (126) is suspended between NS5 in (12345) and the NS5′-brane in $(123[\frac{4}{8}]_\varphi[\frac{5}{9}]_\varphi)$ and intersects with a D5-brane in (12789). Note that the masses of matter with flavors correspond to the position differences in (345)-directions between the D3- and D5-branes. For this theory, all the terms in Eq. (35) are no longer what would be called FI terms, which break either supersymmetry or internal symmetry. Because the second NS5′-brane is rotated in the (8,9)-directions, the 5-brane shifts in these directions can be compensated by the D3-brane shifts in the (4,5)-directions, so that it is possible to preserve both supersymmetry and internal symmetry. On the other hand, the shift in the relative position in the 7-direction of the NS5- and NS5′-branes corresponds to a real FI term.

In $\mathcal{N}=1$ superspace, the mass terms for the hypermultiplet are given by

$$S_M=\int d^3xd^2\theta(\kappa_4X_4^2+\kappa_5X_5^2). \tag{38}$$

In Eq. (38), $|\kappa_4|$ and $|\kappa_5|$ correspond to masses for the scalar fields X_4 and X_5. These masses originate from the relative rotations of the NS5′-brane in the (4, 8)- and (5, 9)-directions. $\mathcal{N}=2$ supersymmetry requires that the masses of the scalar fields should be equal to each other, i.e., $\kappa_4=-\kappa_5\equiv m$. When this relation is satisfied, Eq. (38) can be written as

$$\int d^3xd^2\theta(m\Phi^2+\text{H.c.}), \tag{39}$$

in $\mathcal{N}=2$ superspace. We thus see that the shift of the scalar component of Φ [corresponding to the D3-brane shifts in the (4,5)-directions] cancels the second terms (linear in Φ) in Eq. (35) and also produces mass terms for the hypermultiplets from the last term in Eq. (34). As a result, there exists a phase in which the gauge symmetry is unbroken. This is what we mean when we say that the second terms in Eq. (35) are not FI terms, and is consistent with our above brane picture.

Since the only mass scale in this theory is g^2, we see, from the action (34) with Eq. (39), that the mass of the chiral multiplet Φ is given by $|mg^2|$ and the hypermultiplets Q, $\tilde{Q}$

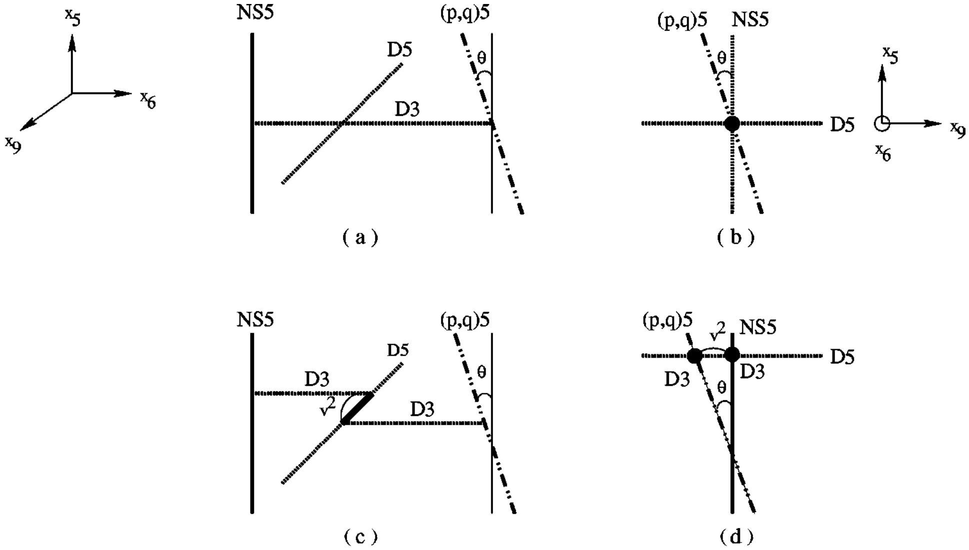

FIG. 2. Topological vortices in Maxwell Chern-Simons theory. (a) or (b) is for the Coulomb phase and (c) or (d) is a vortex solution in the Higgs phase.

and the vector multiplet V are massless. For simplicity, we take the rotation angle $\varphi=\pi/2$. Since then the mass of the chiral multiplet Φ goes to infinity, the chiral field Φ decouples from the theory and can be set to zero, but this simplification does not affect our results.

If we turn on the FI coupling for the vector multiplet by a position difference between the NS5- and NS5$'$-branes in the 7-direction, this introduces a linear superpotential in the action (34) like

$$S_{FI}=-v^2\int d^3x d^4\theta V. \tag{40}$$

From the actions (34) and (40), the scalar potential U can be easily read off as

$$U=\frac{g^2}{2}(|q|^2-|\tilde{q}|^2-v^2)^2+X_3^2(|q|^2+|\tilde{q}|^2). \tag{41}$$

The potential U allows only a symmetry broken vacuum:

$$|q|=v, \quad |\tilde{q}|=0, \quad X_3=0. \tag{42}$$

It is well known that the symmetry broken vacuum (42) admits topological Nielsen-Olesen vortices [24,25,13], where it is shown that the mass of n vortices is $2\pi v^2 n$ and the number of zero modes is $2n$, corresponding to the positions of n vortices. Now we will identify these BPS solutions with the type IIB brane configurations.

Consider the Higgs branch (42) of the model (34) sketched in Figs. 1(c) and 1(d). As shown in the figure, the right-hand NS$'$-brane is shifted by v^2 along the 7-direction. This shift introduces the FI D-term (40). Let us further consider additional D-strings extended to the 7-direction together with the brane configuration in Fig. 1(c). Since the D-strings can end on the D3-branes, we can obtain D-strings with finite length, which means finite energy. We have shown in Sec. II that these D-strings preserve half the supersymmetry, and so should correspond to BPS states. Here we claim that we can identify the D-strings with the topological Nielsen-Olesen vortices in Maxwell-Higgs theory. The vorticity n is just the number of D-strings. Since v^2 has the dimension of mass and is related to the tension of the stretched D-string, we can interpret it as the mass of a vortex. From the brane configuration in Fig. 1(c), we see that the strings can freely move on the (1, 2)-plane, so the translational zero modes of the n D-strings are $2n$. Thus our identification is consistent with the field theory results.

Note that the chiral multiplet Φ is neutral with respect to the gauge group U(1) although it is charged under the $U(1)_{4,5}$ rotation group in the $(x^4,\ x^5)$-directions. Thus the presence of Φ in the theory does not seriously change the story on the existence of the soliton solution. If we set $\kappa_4=-\kappa_5\equiv m=0$, the vortex solutions obtained above can also be considered as the BPS vortices in $\mathcal{N}=4$ theory. This class of solutions should be obtained from the BPS solutions in four-dimensional $\mathcal{N}=2$ QED.

B. Topological and nontopological Maxwell Chern-Simons vortices

Here we will analyze the brane configurations 2(ii) in Table I with and without an additional $\widetilde{\mathrm{M2}}$-brane extended to (2, 9)-directions. The corresponding type IIB brane configurations are depicted in Fig. 2.

Let us identify the three-dimensional $\mathcal{N}=2$ supersymmetric field theories realized on the D3-brane. Consider first the configuration in Fig. 2(a) in which one D3-brane in the direction (126) is suspended between an NS5 in (12345) and a (p,q)5-brane in $(1234[\frac{5}{9}]_\theta)$ and intersects with the N_f D5-branes in (12789). On this configuration, we get U(1) gauge theory with massless N_f flavors in the fundamental representation with no FI terms. (Here we will take $N_f=1$ for simplicity.) Note that the masses of hypermultiplets correspond to the position differences in the (345)-directions between the D3-brane and the D5-branes and the FI terms come from the relative positions in the (78)-directions of the NS5-brane and the (p,q)5-brane. These FI terms are those with the coefficient w^2 in Eq. (35). In fact, we will see that theories only with the first term have a symmetry unbroken phase.

In $\mathcal{N}=2$ superspace [12], the vector multiplet V is composed of A_μ and X_5 corresponding to the A_3 component of the four-dimensional gauge field and the chiral multiplet Φ contains X_3 and X_4, which correspond to strings describing fluctuations of the D3-brane in the transverse directions $(x^3,\ x^4)$. There are also hypermultiplets consisting of Q and $\tilde{Q}$ in the fundamental representation. Similarly to Maxwell-Higgs theory in Eq. (34), we can write down the $\mathcal{N}=2$ supersymmetric action in the Coulomb branch for the brane configuration in Fig. 2(a) [13]:

$$S_{N=2}=\frac{1}{g^2}\left[\int d^3xd^4\theta\Phi^\dagger\Phi+\frac{1}{2}\left(\int d^3xd^2\theta W^\alpha W_\alpha+\text{H.c.}\right)\right]$$
$$+\int d^3xd^4\theta(Q^\dagger e^{2V}Q+\tilde{Q}e^{-2V}\tilde{Q}^\dagger)$$
$$+\frac{1}{\sqrt{2}}\left(\int d^3xd^2\theta\tilde{Q}\Phi Q+\text{H.c.}\right)$$
$$-\frac{1}{2}\int d^3xd^2\theta(\kappa_0U^\alpha W_\alpha-\kappa_5X_5^2). \qquad (43)$$

In Eq. (43), κ_0 and κ_5 correspond to masses for the gauge field A_μ and the scalar field X_5. These masses originate from the simultaneous rotations of the (p,q)5-brane in the (2, ♮)- and (5, 9)-directions. The $\mathcal{N}=2$ supersymmetry requires that the masses of the gauge field and the scalar field should be equal to each other,[3] i.e., $\kappa_0=\kappa_5\equiv\kappa$. Since we set the hypermultiplet masses and FI couplings to zero, the only mass scale in this theory is g^2. In fact, from the action (43), we see that the mass of the vector multiplet V is given by $|\kappa g^2|$ and the hypermultiplets $Q,\ \tilde{Q}$ and the chiral multiplet Φ are massless.[4] For simplicity, we fix the location of the D3-brane in the (3, 4)-plane and put the chiral multiplet Φ to zero.

If we shift the positions between the NS5- and (p,q)5-branes in the 9-direction, this introduces a linear term in the action (43):

$$S_{FI}=-v^2\int d^3xd^4\theta V. \qquad (44)$$

[3]Note that the coupling constant $\kappa=-(1/g_s)\tan\theta=-p/q$ is dimensionless, so our κ corresponds to κ/g^2 in [9,13,16]. In Refs. [9,13], the supersymmetric pure Chern-Simons system was obtained by taking the limit $\kappa\to\infty$ with the ratio κ/g^2 fixed. In our notation, their limit corresponds to $\kappa=$fixed and $g^2\to\infty$. Since $1/g^2=L_6/g_s$, supersymmetric pure Chern-Simons theory can be obtained by taking the limit $L_6\to 0$. As discussed in Ref. [7], since x^6-dependent Kaluza-Klein modes can be ignored as long as $\kappa \ll 1/g_s$, the low-energy approximation in the three-dimensional gauge theory should be valid if the string coupling constant g_s is sufficiently small.

[4]Here and in what follows, we omit factors like $1/4\pi$ associated with κ for simplicity.

From the actions (43) and (44), the scalar potential U can be easily read off as

$$U=\frac{g^2}{2}(|q|^2-|\tilde{q}|^2-v^2+\kappa X_5)^2+X_5^2(|q|^2+|\tilde{q}|^2). \qquad (45)$$

The potential U allows both symmetry broken and unbroken vacua:

$$\text{symmetry broken phase:}\quad |q|=v,\quad |\tilde{q}|=0,\quad X_5=0; \qquad (46)$$

$$\text{symmetry unbroken phase:}\quad |q|=|\tilde{q}|=0,\quad X_5=\frac{v^2}{\kappa}. \qquad (47)$$

Notice that the linear term (44) allows a symmetry unbroken vacuum and this agrees with the brane picture that the shift of the 5-branes in the 9-direction is compensated by the D3-brane shift in the 5-direction. It is well known [13,16] that the symmetry unbroken phase admits nontopological BPS multisoliton solutions, while the symmetry broken phase admits topological BPS multisoliton solutions. Now our next goal is to find the type IIB brane realizations for these soliton solutions.

1. Topological vortices

First we go to the Higgs branch (46) of the model (43) sketched in Fig. 2(c). As shown in the figure, the right-hand D3-brane is slid by v^2 along the 9-direction. This introduces the FI D-term (44). Since the transverse fluctuations of the D3-branes along the (3, 4)-plane are highly suppressed, the chiral field Φ decouples from the theory and can be set to zero. In this Higgs branch, the theory is mapped to the $\mathcal{N}=2$ Maxwell Chern-Simons theory studied in Ref. [13], where the mass of n vortices is $2\pi v^2 n$ and the number of zero modes is $2n$ corresponding to the positions of n vortices.

Let us consider the D-strings extended to the 9-direction together with the brane configuration in Fig. 2(c). We can now apply the similar logic as that in Sec. III A. Since the D-strings ending on the D3-branes have a finite length, the D-strings have a finite energy proportional to their length. We have shown in Sec. II that these D-strings preserve half the supersymmetry, and so correspond to BPS states. Thus we can identify the D-strings, i.e., (0,1)-strings, with the topological vortices in the Maxwell Chern-Simons theory. Note that, in the presence of the axion field χ, the tension formula (33) implies that the vortex also carries electric charge Q_e proportional to magnetic charge Q_m, i.e. $Q_e=\kappa Q_m$, as an analogue of Witten's effect. Field theoretically, this is just the Gauss law constraint [15,16]. The vorticity n is just the number of D-strings. Since v^2 has the dimension of mass and is related to the energy of the stretched D-string, we can also interpret it as the mass of a vortex. From the brane configuration in Fig. 2(c), we see that the strings can freely move on the (1, 2)-plane, so the translational zero modes of the n D-strings are $2n$. Thus our identification is consistent with the field theory results [16,25,13].

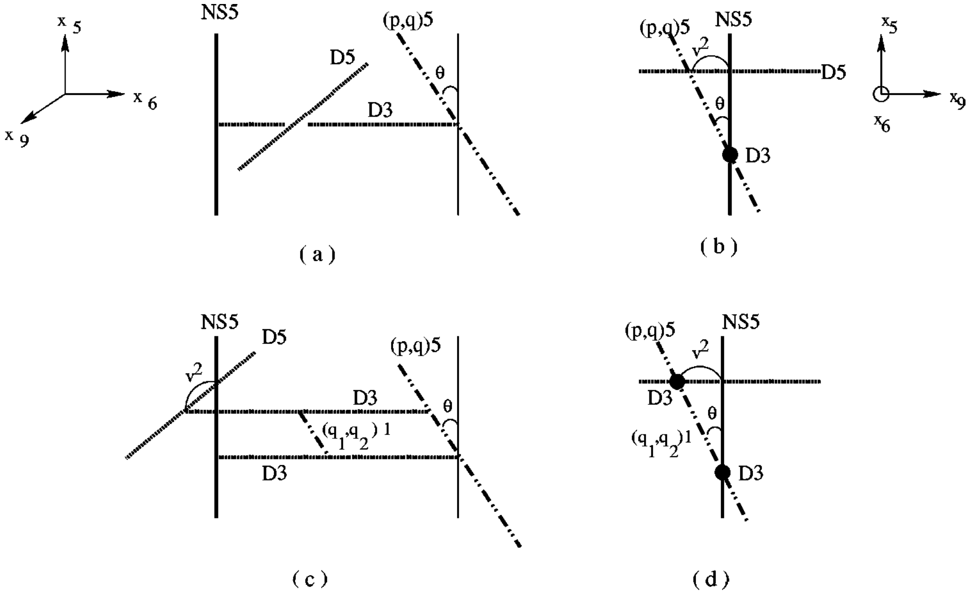

FIG. 3. Symmetric phase (a) or (b) and nontopological vortices (c) or (d).

2. Nontopological vortices

Next we consider the symmetry unbroken phase (47). We summarize the corresponding brane configuration in Fig. 3(a). The D5-brane is lifted up along the 5-direction by v^2/κ relative to the D3-brane, which gives the field X_5 a vacuum expectation value as in Eq. (47). Then Eq. (45) shows that it also induces the mass $m=|v^2/\kappa|$ to the hypermultiplets Q and $\tilde{Q}$.

Here we will focus on the $b=5$ case out of the $\widehat{\mathrm{M2}}$-branes constructed in Sec. II which may correspond to the nontopological vortices in theories listed as 2(ii) in Table I. Upon compactification, the $\widehat{\mathrm{M2}}$-brane reduces to either the F-string in the $[\frac{5}{9}]_\theta$-direction or the D2-brane in the $(2[\frac{5}{9}]_\theta)$-directions depending on their world volumes, and a further T_2-dual transformation gives the F1$\oplus$D1 bound state in the $[\frac{5}{9}]_\theta$-direction. Since the $\widehat{\mathrm{M2}}$-brane is rotated from $x^{\natural}$ by θ in the $(2,\ \natural)$-plane, the number (q_1,q_2) characterizing the (F1,D1) bound state satisfies the relation:[5] $g_s^{-1}\tan(\pi/2+\theta)=-q_1/q_2$. The (F1,D1) bound state has been studied in Ref. [2] where it has been shown that for the configuration of parallel F-string and D-string, the F-string dissolves in the D-string, leaving flux behind and the resulting bound state, i.e., D-string with flux, is supersymmetric. In addition, it has been shown that there is a bound string saturating the BPS bound for all (q_1,q_2) with relatively prime q_1 and q_2, named a (q_1,q_2)-string [21]. In the presence of (q_1,q_2)-string, the axion field in Eq. (33) is shifted by $-q_1/q_2$, that is, $\chi=-\kappa-q_1/q_2$. Thus the nontopological vortex also carries electric charge Q_e proportional to magnetic charge Q_m, i.e., $Q_e=\kappa Q_m$. All these properties are consistent with those in field theory results [15,16].

Here we suggest the type IIB brane configuration in Fig. 3(c) as a possible candidate for the nontopological BPS vortices, where they are represented by D-strings with fluxes connecting two D3-branes.[6] From field theory [13,16,25], we know that the magnetic flux Φ and electric charge Q of the nontopological vortices are not quantized, $\Phi=-Q/\kappa=2\pi(n+\alpha)$, where n is the vorticity of the solitons and $\alpha\geqslant n+2$ is an undetermined parameter characterizing the asymptotic behavior of the solutions. It was shown that the number of zero modes in the nontopological soliton background is $2n+2\hat{\alpha}-2$, where $\hat{\alpha}$ is the greatest integer less than α. We interpret the number n as the number of D-strings with fluxes since they can freely move on the (1, 2)-plane, so the translational zero modes of the n D-strings are[7] $2n$. In Ref. [16], $2\hat{\alpha}-2$ is interpreted as the moduli parameters specifying the fluxes and the U(1) phases of lumps. If our identification is correct, it should be related to the moduli parameters of F-string fluxes.

3. $\mathcal{N}=3$ theories

Next let us consider BPS vortices in $\mathcal{N}=3$ theories [8,9]. The supersymmetry analysis in Sec. II shows that the $\widetilde{\mathrm{M2}}$-brane also corresponds to a BPS vortex solution preserving 1/16 supersymmetry in $\mathcal{N}=3$ theory. $\mathcal{N}=3$ Maxwell Chern-Simons theory was considered in Ref. [9] and the action can be obtained from Eq. (43) by further adding mass terms for the chiral multiplets coming from the rotations of

[5]Note the difference that (p,q) corresponds to (D5,NS5) charges whereas (q_1,q_2) to (F1,D1) charges, respectively. That this is correct can be understood from the fact that $\theta=\pi/2$ $(q_1=0)$ gives a pure D-string in type IIB theory.

[6]The dynamics of the upper D3-brane in Fig. 3(c) or 3(d) is not gauge theory but dual scalar theory. This brane only serves as a boundary state invisible in the lower D3-brane. On the other hand, the theory of the lower D3-brane is just our U(1) Maxwell Chern-Simons gauge theory.

[7]Although Fig. 3(c) shows that the vortices can move along the x^6, the moduli of this motion are massive since x^6 has a finite interval.

(3, 7)- and (4, 8)-planes and FI couplings of the type (35). The scalar potential U for the $\mathcal{N}=3$ case is given by

$$U=\frac{g^2}{2}(|q|^2-|\tilde{q}|^2-v^2+\kappa X_5)^2+\frac{g^2}{2}|q\tilde{q}+\kappa\phi-w^2|^2 +(|q|^2+|\tilde{q}|^2)(X_3^2+|\phi|^2). \tag{48}$$

This potential allows a symmetry broken as well as unbroken vacua:

$$\text{symmetry broken phase:}\quad |q|^2-|\tilde{q}|^2=v^2,\quad q\tilde{q}=w^2,\quad \phi=X_5=0; \tag{49}$$

$$\text{symmetry unbroken phase:}\quad |q|=|\tilde{q}|=0,\quad \phi=\frac{w^2}{\kappa},\quad X_5=\frac{v^2}{\kappa}. \tag{50}$$

The brane configurations for the BPS vortices in $\mathcal{N}=3$ theory, for example, in case 4(iii) in Table I are essentially the same as those in Figs. 2 and 3. The D-strings in the asymmetric phase (49) and (q_1,q_2)-strings in the symmetric phase (50) correspond to the topological and nontopological vortices, respectively, constructed in Ref. [9] [where the potential U is of the case $w^2=0$ in Eq. (48)].

As discussed in footnote 3, the vortex solutions of the $\mathcal{N}=2$ and $\mathcal{N}=3$ supersymmetric Chern-Simons systems considered in Refs. [11,8] could be obtained by taking the limit $L_6\to 0$ with κ fixed from $\mathcal{N}=2$ and $\mathcal{N}=3$ Maxwell Chern-Simons theory, respectively.

IV. DISCUSSIONS

In this paper we have considered M-brane configurations which can be reduced to type IIB branes corresponding to BPS solitons in three-dimensional gauge theories. In a given M-brane background preserving $\mathcal{N}=4$, 3, 2 supersymmetry, we have found BPS M2-branes preserving $\mathcal{N}=2,1$ supersymmetry, where the $\mathcal{N}=2$ case is obtained only for $\mathcal{N}=4$ theory and we have identified the brane configurations with soliton spectra of the field theories. Although our construction via the type IIB branes can achieve nice agreements with the vortex solutions of field theory, the type IIB brane construction of BPS domain wall solutions remains an open problem. We will briefly discuss BPS M2-branes which are plausible candidates for the BPS domain wall solutions in field theory.

The type IIB brane construction in this paper will be universally valid provided that $L_6<l_s<1/\kappa g^2$. Thus supersymmetric Chern-Simons theories can be obtained by taking the limit $L_6\to 0$ from Maxwell Chern-Simons theories with a Chern-Simons coupling κ fixed, since, in the limit, the mass of the gauge boson becomes infinite and so the kinetic Maxwell term is decoupled. Note, in the limit, that, in the case of Maxwell theory without a Chern-Simons term, the gauge boson remains massless so that the vector multiplet does not decouple; i.e., the theory flows only to the strong coupling limit. On the other hand, when $\kappa\to\infty$ in $\mathcal{N}=3$ theory, the vector multiplet completely decouples and the supersymmetry is enhanced to $\mathcal{N}=4$. In the limit, the theory flows to a free theory of massless hypermultiplets [6].

In three-dimensional Maxwell Chern-Simons theory, it is known that there can be BPS domain wall solitons [16,17]: topological domain walls interpolating the symmetric and asymmetric phases, and nontopological domain walls residing in the symmetric phase. The domain walls constructed in field theory are finite energy density solutions. As also noted in Sec. II, we can introduce the M2′-brane preserving the supersymmetry and extended to (5, 9)-directions. In type IIB string theory, this brane will be a D3-brane extended in the (2, 5, 9)-directions and, in three-dimensional field theory, this will correspond to a one-dimensional object extended along the x^2-direction (maybe a domain wall). However, there are some problems in the solution. First of all, the brane configurations do not give finite energy density solutions. For such solutions, we need D3-branes with finite area in the (5, 9)-plane. Next, in the cases of $\mathcal{N}=1$ and $\mathcal{N}=3$ theory, there is no explicitly known BPS domain wall solution in field theory whereas the supersymmetry analysis in Sec. II shows that the M2′-brane preserves fractional supersymmetry. However, note that the domain wall solution can be reduced to a two-dimensional field theory solution as in [17]. Then the $\mathcal{N}=1$ or $\mathcal{N}=3$ theory corresponds to the two-dimensional $\mathcal{N}=(1, 1)$ or $\mathcal{N}=(3, 3)$ supersymmetry, respectively. Thus, even in these cases, the M2′-brane may correspond to BPS states in the sense of two-dimensional field theory.

Nevertheless, let us speculate on possible brane configurations for the topological and nontopological domain walls. Consider the configurations in Figs. 2(c) and 3(a) altogether. The resulting configurations are sketched in Fig. 4. In Sec. II, we have shown that there can be a BPS state represented by a D3-brane extended along the (2, 5, 9)-directions. The desired solution is D3-branes confined along the 5- and 9-directions to obtain a finite energy density solution. If we could have a D3-brane solution with finite area such as the triangle in Fig. 4(b) or 4(d), its energy density $\mathcal{E}$, energy per unit length, will be given by the area of the triangle, i.e., $\mathcal{E}=v^4/\kappa$, which is coincident with the field theory result [16,17].

If it is correct, according to the classification in Refs. [16,17], the solution depicted in Fig. 4(a) or 4(b) will correspond to a topological BPS domain wall since the D3-brane is interpolating the symmetric and asymmetric phases. On the other hand, the solution in Fig. 4(c) or 4(d) will correspond to a nontopological domain wall since it is residing only in the symmetric phase.

Unfortunately, it seems that such a D3-brane solution in type IIB supergravity realizing the BPS domain wall solution with finite energy density has not been known until now. So at the moment it is difficult to speculate on the problem more precisely. It will be interesting to look at these problems more closely from both the field theory side and in string theory side.

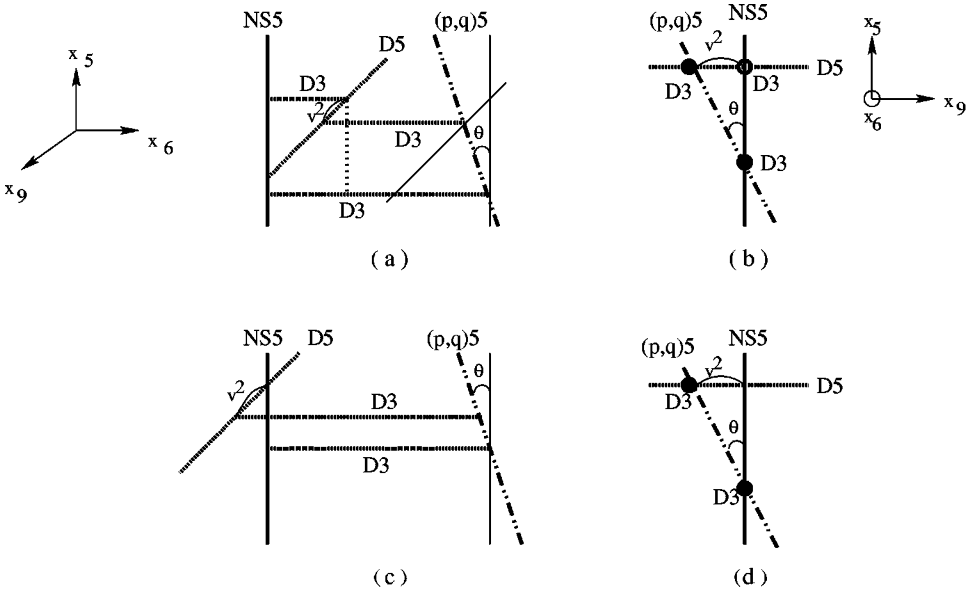

FIG. 4. Possible domain walls in Maxwell Chern-Simons theory. (a) or (b) is topological and (c) or (d) is nontopological.

Finally let us briefly discuss mirror symmetry [3–6] for soliton spectra. Since the mirror symmetry is obtained from the SL(2,**Z**) transformation S of type IIB string theory and a rotation R that maps the x^i to x^{i+4} $(i=3,\ 4,\ 5)$, the combined operation RS exchanges NS-branes (e.g., NS5-brane and F-string) to D-branes (e.g., D5-brane and D-string) and maps D3-brane to itself. Moreover, the mirror map exchanges the Higgs and Coulomb branches. Thus the mirror symmetry transforms a D-string corresponding to a soliton in the Higgs phase into an F-string corresponding to a fundamental particle in the Coulomb phase. This means that the mirror symmetry exchanges particles and vortices [6,5]. As discussed in Refs. [7,6], mirror symmetry transforms Maxwell Chern-Simons theory into a self-dual model with Chern-Simons coupling $\kappa' = -1/\kappa = q/p$ in Ref. [26]. If the mirror symmetry is exact in $\mathcal{N}=2$ or $\mathcal{N}=3$ theory, our vortex construction shows that the nontopological vortices have to transform into those of the self-dual model represented by $(q_2, -q_1)$-strings. Thus we expect the mirror map in Maxwell Chern-Simons theory will have a more rich spectrum. It will be interesting to explicitly investigate mirror symmetry and the brane creation in Refs. [3,7] including soliton sectors in the theory.

In this paper we have only considered Abelian gauge theories. However, one may also construct the non-Abelian Yang-Mills Chern-Simons theory and its Higgs phase via type IIB brane configurations. (The $\mathcal{N}=3$ supersymmetric non-Abelian Chern-Simons theory and its breaking to $\mathcal{N}=2$ is partially constructed in [19].) We hope that a generalization of the present work will be achieved in the near future.

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[1] See, for example, A. Giveon and D. Kutasov, hep-th/9802067 and references therein.
[2] E. Witten, Nucl. Phys. **B460**, 335 (1996).
[3] A. Hanany and E. Witten, Nucl. Phys. **B492**, 152 (1997).
[4] J. de Boer, K. Hori, H. Ooguri, Y. Oz, and Z. Yin, Nucl. Phys. **B493**, 148 (1997); J. de Boer, K. Hori, Y. Oz, and Z. Yin, *ibid.* **B502**, 107 (1997).
[5] O. Aharony, A. Hanany, K. Intriligator, N. Seiberg, and M. J. Strassler, Nucl. Phys. **B499**, 67 (1997).
[6] A. Kapustin and M. J. Strassler, J. High Energy Phys. **04**, 021 (1999).
[7] T. Kitao, K. Ohta, and N. Ohta, Nucl. Phys. **B539**, 79 (1999).
[8] H. Kao and K. Lee, Phys. Rev. D **46**, 4691 (1992).
[9] H. Kao, Phys. Rev. D **50**, 2881 (1994).
[10] P. di Vecchia and S. Ferrara, Nucl. Phys. **B130**, 93 (1977).
[11] C. Lee, K. Lee, and E. Weinberg, Phys. Lett. B **243**, 105 (1990).
[12] E. A. Ivanov, Phys. Lett. B **268**, 203 (1991).
[13] B.-H. Lee, C. Lee, and H. Min, Phys. Rev. D **45**, 4588 (1992); B.-H. Lee and H. Min, *ibid.* **51**, 4458 (1995).
[14] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[15] J. Hong, Y. Kim, and P. Y. Pac, Phys. Rev. Lett. **64**, 2230 (1990); R. Jackiw and E. Weinberg, *ibid.* **64**, 2234 (1990).
[16] R. Jackiw, K. Lee, and E. Weinberg, Phys. Rev. D **42**, 3488 (1990).
[17] H. Kao, K. Lee, and T. Lee, Phys. Rev. D **55**, 6447 (1997); P. K. Tripathy, *ibid.* **57**, 5166 (1998).
[18] N. Ohta and P. K. Townsend, Phys. Lett. B **418**, 77 (1998).

[19] K. Ohta, J. High Energy Phys. **06**, 025 (1999).
[20] G. Papadopoulos and P. K. Townsend, Phys. Lett. B **380**, 273 (1996); A. A. Tseytlin, Nucl. Phys. **B475**, 149 (1996); J. P. Gauntlett, D. A. Kastor, and J. Traschen, *ibid.* **B478**, 544 (1996); R. Argurio, F. Englert, and L. Houart, Phys. Lett. B **398**, 61 (1997); N. Ohta, *ibid.* **403**, 218 (1997).
[21] J. H. Schwarz, Phys. Lett. B **360**, 13 (1995).
[22] E. Witten, Phys. Lett. **86B**, 283 (1979).
[23] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd ed. (Princeton University Press, Princeton, 1992).
[24] E. Weinberg, Phys. Rev. D **19**, 3008 (1979); **24**, 2669 (1981).
[25] C. Lee, H. Min, and C. Rim, Phys. Rev. D **43**, 4100 (1991).
[26] P. K. Townsend, K. Pilch, and P. van Nieuwenhuizen, Phys. Lett. **136B**, 38 (1984); **137B**, 443 (1984); S. Deser and R. Jackiew, *ibid.* **139B**, 371 (1984).